Diversity and the Limits of Reason

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Gödel's Incompleteness Theorems imply that diversity must be a complement to reason for unbounded creativity. Understanding this is crucial in this age of globalization.

Introduction

Logic discovered its own limitations in Gödel's Incompleteness Theorems. Before Gödel's proofs, mathematicians sought a logical mechanistic process that could, with enough time and resources, decide any mathematical question. Gödel proved this was impossible. Any consistent sufficiently powerful set of mathematical assumptions or axioms can be expanded in simple obvious ways and complex ones. The standard example is to add the axiom that the system itself is consistent. Gödel proved this cannot be derived *within* any consistent system strong enough to define an ideal computer. An ideal computer can run forever error free and has access to unlimited storage.

Any single path approach to expanding mathematics will encounter a Gödelian limit.[1] Progress can be made forever, but all the results, over an infinite time, are subsumed in a single axiom that will never be discovered. Only a divergent process that explores an ever increasing number of alternatives, providing increasing resources to each viable path, can escape a Gödelian limit. There is no way to select between alternative expansions of mathematics beyond eliminating those that are inconsistent or proved wrong for more complex reasons. For example, a contradiction exists only if two consequences are inconsistent, but there may still be an infinite sequence of consequences that are mutually inconsistent.

The proof that all mathematical truth can be explored in this way is trivial. If resources are available, all alternatives can be considered, each with ever greater effort. There is evidence that this is relevant to the evolution of the mathematically capable human mind, to the future expansion of mathematics and to cultural creativity in general.

[1]Roger Penrose has argued that quantum effects in the brain allow mathematicians to transcend the limitation of Gödel's proof[7]. This is not necessary to explain the mathematically capable human mind. All that requires is the enormous diversity of biological evolution as discussed below. There is no significant evidence to support Penrose's idea.

Mathematical Truth

Mathematicians have not adopted a divergent approach to expanding mathematics. The dominant formalization of mathematics is the Zermelo Frankel axioms of set theory plus the Axiom of Choice (ZFC)[3, 1]. ZFC seems adequate for all practical problems in science and engineering and most open mathematical questions[5]. This approach to mathematics is justified by a Platonic philosophy that assumes the existence, in some abstract sense, of a hierarchy of infinite sets which encode mathematical truth that cannot be decided by finite processes. Some mathematicians are questioning this philosophy and the objectivity of some open problems in mathematics that depend on it, most prominently the Continuum Hypothesis. This question arose from the proof that there must be *more* real numbers than integers because one cannot pair up *every* real number with a *unique* integer. This is obviously correct for finite sets but can be questioned for infinite ones. The Continuum Hypothesis asserts that the reals are the smallest set larger than the integers. Both the Continuum Hypothesis and its negation have been shown to be consistent with ZFC[3]. Thus the Continuum Hypothesis can never be proved true or false within ZFC.

Solomon Feferman, the editor of Gödel's collected works, expressed his skepticism as follows.

> I am convinced that the Continuum Hypothesis is an inherently vague problem that *no* new axiom will settle in a convincingly definite way. Moreover, I think the Platonic philosophy of mathematics that is currently claimed to justify set theory and mathematics more generally is thoroughly unsatisfactory and that some other philosophy grounded in inter-subjective human conceptions will have to be sought to explain the apparent objectivity of mathematics.
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> [Feferman's note to the above quote] CH [Continuum Hypothesis] is just the most prominent example of many set-theoretical statements that I consider to be inherently vague. Of course, one may reason confidently *within* set theory (e. g., in ZFC) about such statements *as if* they had a definite meaning[5].

I have argued for a philosophy of mathematical truth that limits objectively meaningful mathematical questions to those relevant to ultimate destiny in a finite but potentially infinite universe. Such questions are logically determined by a list of events that an ideal computer could enumerate[2]. I am skeptical of infinite structures in the physical universe or in an ideal Platonic abstract reality. The concept of an ideal Platonic reality is firmly rooted in the physical universe. For example, we cannot construct the Platonic perfect circle, but we can compute the ratio of its circumference to its diameter, π, to millions of decimal places with high confidence that we have done it correctly. If we cannot reach Platonic perfection, we can often approach it to an arbitrary accuracy through technology. Nothing in physical reality seems able to approach the infinite structures in the Platonic philosophy underlying ZFC.

Creativity and Evolution

Nervous systems and sense organs evolved to control an organisms actions taking into account likely consequences. Reason, logic and the capacity for mathematics evolved because they were useful in determining likely outcomes. Central to mathematics is induction on the integers and its generalizations. Induction allows one to derive a potentially infinite number of conclusions from a single proof. Mathematical induction is a product of cultural evolution, but the mind that can think about iterative and self reflective generalizations is a product of biological evolution. Gödel's result established that there is no finite limit to the generalizations of induction that are theoretically useful in predicting the consequences of ones actions.

The levels of induction derivable from ZFC seem adequate for every practical problem we currently encounter.[2] A Gödelian limit must be something it is hard to see beyond, so it is not surprising that we do not see a clear need to extend the foundations of mathematics. However, I suspect ZFC is far from the Gödelian limit of the human mind combined with the culture and technology that mind can lead to. A philosophy of mathematics that focuses on the combinatorial structure of axioms and uses computers to do experiments on that structure may lead to much stronger generalizations of induction than can be derived from ZFC.

Evolution faced many major obstacles as it evolved the extraordinarily complex human brain. Some of these could be interpreted as Gödelian limits. They were only overcome by the immense diversity of life, the enormous time that was available and perhaps some good luck. The end of the age of dinosaurs is one example. They dominated the earth far longer than we have. They would probably still be dominant if not for that terrible wonderful meteor that destroyed them and a large percentage of all species. The world dominated by dinosaurs was less diverse then the planet repopulated with small mammals. A dinosaur dominated planet may not have had the diversity needed to evolve a mathematically capable brain. The human mind is the best example of overcoming a Gödelian limit. The structure of that brain has given us the capacity to create technology and transform the planet.

Unlimited creativity, at least in mathematics, requires both ever more resources for each viable path and an ever increasing number of paths. These are exemplified in the diversity of life and the complexity of the human brain. The inherent conflict in these two requirements is reflected in cultural and biological evolution. Carl Jung, in defining the modern usage of the psychological terms, intravert and extravert, observed that it applies to these psychological dispositions and to the fundamental strategies for reproductive success.

> There are in nature two fundamentally different modes of adaptation which ensure the continued existence of the living organism. The one consists of

[2]There are classes of what might be considered practical problems, like the computer halting problem, that no finite formal system can solve for all examples. The computer halting problem asks if an ideal computer with a specific program will run forever or eventually halt. It is straight forward to construct a computer program that will halt if and only if the existing formalization of mathematics is inconsistent.

> a high rate of fertility, with low powers of defense and short duration of life for the single individual; the other consists in equipping the individual with numerous means of self-preservation plus a low fertility rate. This biological difference, it seems to me, is not merely analogous to, but the actual foundation of, our two psychological modes of adaptation [intraversion and extraversion][6, ¶559]

Jared Diamond in *Guns, Germs and Steel*[4] observed a similar creative dialectic between diversity and concentration of resources in cultural evolution. He investigated why certain cultures came to dominate the planet while others remained relatively stagnant. One needed an appropriate balance between diversity and concentration of resources for modern civilization to arise. A culture dominated by a single ruling elite, like China, inevitably failed to pursue possibilities essential to future development. In contrast, a region, like Africa, with so many small communities, could never marshal the resources needed for certain kinds of progress. Europe presented the ideal combination of diversity and concentration of resources.

There are many reasons why a good tradeoff between diversity and concentration of resource is important.

- Either quantum mechanics is incomplete or the laws of physics are irreducibly random.[3]
- Many important phenomena, like hurricanes, are chaotic or chaotic like.[4]
- The ability to predict the consequences of choices is limited by knowledge of initial conditions and the practical capacity to work out the implications of what is known.
- Today the imperfection of human institutions is the most important reason diversity is essential. Government and economic institutions often create perverse incentives and select poor leaders. If a handful of badly structured and/or lead companies dominate the economy or a handful of badly governed countries dominate the planet, there will not be the competition that forces change.

The above reinforce the need for diversity. If you do not know what is going to happen or how to create large institutions, hedge your bets. The problem of creating effective human institutions may ultimately require new mathematics. It involves the capabilities of the human mind and how those minds interact in a particular culture. In the near term, expansion of mathematics is unlikely to help us make better decisions. However, no matter how much progress we make in predicting and controlling our world through the exponential expansion of technology, expanding diversity will remain an irreducible

[3]Quantum randomness is a closed question for the majority of physicists, but a minority, including the 1999 Nobel prize winner, Gerard t Hooft[8], are unconvinced.

[4]If the fundamental laws of physics turn out to be discrete and not continuous, as many prominent physicists suspect, there are no true chaotic processes, but discrete models can simulate the effects of chaos to a high enough accuracy that there may be no practical difference in most circumstances.

logical requirement for unlimited creativity. The limits of mathematics will be of practical importance eventually, if we are able to survive the immediate problems we are creating for ourselves.

Globalization and Diversity

Technology makes globalization possible through communications and transportation. It is making it necessary because of the threats to the environment, especially global warming, it has created. We need a limited global unity to preserve the environment and establish the peaceful just societies essential for human progress. Any unity is a potential threat to diversity. I suspect we will be unable to achieve the essential unity in the absence of institutions that sufficiently foster diversity. The creative instincts may be too deeply imbued in our genes and psyche.

We can become united in preserving the environment and world peace while continuing to expand diversity as resources permit,[5] but it will not be easy. Technology is changing the world far faster then biological evolution can adapt. This makes reason and the objectivity of empiricism of ever greater importance. Crucial to using these tools is to recognize the limitations they have established for themselves.[6]

References

[1] Paul Budnik. *What is and what will be: Integrating spirituality and science*. Mountain Math Software, Los Gatos, CA, 2006. 2, 5

[2] Paul Budnik. What is Mathematics About? *Philosophy of Mathematics Education*, 22, 2007. 2

[3] Paul J. Cohen. *Set Theory and the Continuum Hypothesis*. W. A. Benjamin Inc., New York, Amsterdam, 1966. 2

[4] Jared Diamond. *Guns, Germs, and Steel : The Fates of Human Societies*. W.W. Norton & Company, New York, 1999. 4

[5] Solomon Feferman. Does mathematics need new axioms? *American Mathematical Monthly*, 106:99–111, 1999. 2

[5] Of course a finite planet will ultimately impose limits. However we may eventually develop robotic space ships with all our knowledge and the capacity to develop civilization on planets in distant solar systems. The universe may be potentially infinite. Every previous boundary has been greatly expanded. Cosmology is of necessity a highly speculative science that is continually evolving.

[6] The ideas in this paper are more fully developed in *What is and what will be: Integrating spirituality and science*[1]. There is a related video, 'Mathematical Infinity and Human Destiny', on Google Video.

[6] Carl Gustav Jung. *Psychological Types*, volume 6 of *The Collected Works of C. G. Jung*. Princeton University Press, Princeton, New Jersey, 1971. 4

[7] Roger Penrose. *The Emperor's New Mind : Concerning Computers, Minds, and the Laws of Physics*. Penguin, New York, 1991. 1

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